CSE 403

Software Engineering

Winter 2023

Advanced program analysis

A primer on solver-aided reasoning and verification

What is a SAT solver?

What is a SAT solver?

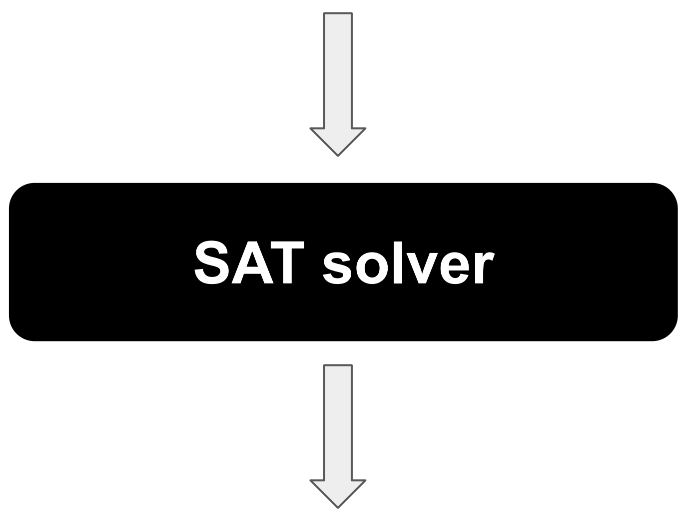
- Takes a **formula** (propositional logic) as input.

$$(X1 \lor X2) \land (\neg X1 \lor X3) \land (X1 \lor \neg X3) \land (\neg X2 \lor \neg X3)$$

What is a SAT solver?

- Takes a **formula** (propositional logic) as input.
- Returns a **model** (an assignment that satisfies the formula).

$$(X1 \lor X2) \land (\neg X1 \lor X3) \land (X1 \lor \neg X3) \land (\neg X2 \lor \neg X3)$$



X = {X1, X2, X3} = **{T, F, T}**

What is Z3?

- An SMT (Satisfiability Modulo Theories) solver.
  - Supports formulas for more complex data types
  - Theories for Integers, Strings, Arrays, etc.

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- An SMT (Satisfiability Modulo Theories) solver.
  - Supports formulas for more complex data types
  - Theories for Integers, Strings, Arrays, etc.
  - Examples for Integers:
    - `a * 1 = a` (identity element)
    - `a + 0 = a` (identity element)

What is Z3?

- An SMT (Satisfiability Modulo Theories) solver.
- Uses a standard language (SMT-LIB).
  - Print to the screen.
  - Declare variables and functions.

```
(echo "Running Z3...")
(declare-const a Int)
```

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  - Define constraints.

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  - Check satisfiability and obtain a model.
  - ...

```
(echo "Running Z3...")
(declare-const a Int)
(assert (> a 0))
(check-sat)
(get-model)
```

Which question does this code answer?

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  - ...

```
(echo "Running Z3...")
(declare-const a Int)
(assert (> a 0))
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```

This code is asking the question:
Does an integer greater than 0 exist?

A first example

```
int simpleMath(int a, int b) {
  assert(b>0);
  if(a + b == a * b) {
    return 1;
  }
  return 0;
}
```

Does this method ever return 1?

A first example

```
int simpleMath(int a, int b) {
  assert(b>0);
  if(a + b == a * b) {
    return 1;
  }
  return 0;
}
```

```
(declare-const a Int)
(declare-const b Int)

(assert (> b 0))
(assert (= (+ a b) (* a b)))

(check-sat)
(get-model)
```

Does this method ever return 1? Let’s ask Z3...

A more complex example

```
int getNumber(int a, int b, int c) {
  if (c==0) return 0;
  if (c==4) return 0;
  if (a + b <  c) return 1;
  if (a + b >  c) return 2;
  if (a * b == c) return 3;
  return 4;
}
```

Does this method ever return 3?
What constraints must be satisfied?

A more complex example

```
int getNumber(int a, int b, int c) {
   if (c==0) return 0;
   if (c==4) return 0;
   if (a + b <  c) return 1;
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}
```

All of the following must be true:

- `!(c == 0)`
- `!(c == 4)`
- `!(a + b < c)`
- `!(a + b > c)`
- `a * b == c`

Does this method ever return 3?

A more complex example

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- `!(c == 0)`
- `!(c == 4)`
- `!(a + b < c)`
- `!(a + b > c)`
- `a * b == c`

(a + b == c) ∧ (a * b == c) ∧ (c != 0) ∧ (c != 4)

A more complex example

```
int getNumber(int a, int b, int c) {
   if (c==0) return 0;
   if (c==4) return 0;
   if (a + b <  c) return 1;
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All of the following must be true:

- `!(c == 0)`
- `!(c == 4)`
- `!(a + b < c)`
- `!(a + b > c)`
- `a * b == c`

```
(declare-const a Int)
(declare-const b Int)
(declare-const c Int)

(assert (not (= c 0)))
(assert (not (= c 4)))
(assert (not (< (+ a b) c)))
(assert (not (> (+ a b) c)))
(assert (= (* a b) c))

(check-sat)
```

A more complex example

```
int getNumber(int a, int b, int c) {
  if (c==0) return 0;
  if (c==4) return 0;
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All of the following must be true:

- `!(c == 0)`
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- `a * b == c`

Z3 supports Bitvectors of arbitrary size.
Let's model Java ints (32 bits) and ask the same question...

A more complex example

```
int getNumber(int a, int b, int c) {
  if (c==0) return 0;
  if (c==4) return 0;
  if (a + b <  c) return 1;
  if (a + b >  c) return 2;
  if (a * b == c) return 3;
  return 4;
}
```

All of the following must be true:

- `!(c == 0)`
- `!(c == 4)`
- `!(a + b < c)`
- `!(a + b > c)`
- `a * b == c`

```
(define-sort JInt () (_ BitVec 32))

(declare-const a JInt)
(declare-const b JInt)
(declare-const c JInt)

(assert (not (= c #x00000000)))
(assert (not (= c #x00000004)))
(assert (not (bvslt (bvadd a b) c)))
(assert (not (bvsgt (bvadd a b) c)))
(assert (= (bvmul a b) c))

(check-sat)
(get-model)
```

Reasoning about program equivalence

```
int add1(int a, int b) {
  return a + b;
}

int add2(int a, int b) {
  return a * b;
}
```

Are these two methods semantically equivalent?

Reasoning about program equivalence

```
int add1(int a, int b) {
  return a + b;
}

int add2(int a, int b) {
  return a * b;
}
```

```
(declare-const a Int)
(declare-const b Int)

(declare-const add1 Int)
(declare-const add2 Int)

(assert (= add1 (+ a b)))
(assert (= add2  (* a b)))
(assert (= add1 add2))

(check-sat)
(get-model)
```

Are these two methods semantically equivalent?

Reasoning about program equivalence

```c
int add1(int a, int b) {
  return a + b;
}

int add2(int a, int b) {
  return a * b;
}
```

```
(declare-const a Int)
(declare-const b Int)

(declare-const add1 Int)
(declare-const add2 Int)

(assert (= add1 (+ a b)))
(assert (= add2  (* a b)))
(assert (= add1 add2))

(check-sat)
(get-model)
```

Yes, for a=2 and b=2.

What have we actually proven here?

Reasoning about program equivalence

```
int add1(int a, int b) {
  return a + b;
}

int add2(int a, int b) {
  return a * b;
}
```

```
(declare-const a Int)
(declare-const b Int)

(declare-const add1 Int)
(declare-const add2 Int)

(assert (= add1 (+ a b)))
(assert (= add2  (* a b)))
(assert (not (= add1 add2)))

(check-sat)
(get-model)
```

For **universal claims**, our goal is to **prove** the absence of counter examples (i.e., the defined constraints are **unsat**)!

Summary

- Solver-aided reasoning is used for testing and verification.
- SMT solvers:
  - Provide one solution, if one exists.
  - Are commonly used to find counter-examples (or prove `unsat`).
  - Support many theories that can model program semantics.
  - Usually support a standard language (SMT-lib).
- The challenge is to model a problem as a constraint system.
  A few examples:
  - Statistical test selection
  - Data-structure synthesis
  - Program synthesis
- Many higher-level DSLs and language bindings exist.